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Technical Notes

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Performance of Electrodynamic Tethers and Ion Thrusters Against Hybrid Systems

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Nomenclature

A = tether cross-sectional area
$B_\perp$ = geomagnetic component perpendicular to the orbital plane
$c_{ch}(c_{ep})$ = jet velocity of chemical (electric) propulsor
E_m = motional field
F = thrust
I_{av} = current averaged over tether length
i_{av} = average current normalized with short-circuit current
L = tether length
L_{bt} = length of tether segment left bare
L^* = length characterizing bare-tether collection
$\dot{m}$ = mass flow rate
N_0 = density of ambient electrons
p = tether cross-sectional perimeter
U_{orb} = orbital velocity
V = bias of ion thruster
W_m = magnetic (thrust) power
W_s = supply power
Z = tether resistance
α = inverse specific power of supply plant
α_t = tether-related hardware factor
Δv = velocity increment of payload
η_{eff} = overall tether efficiency
η_{hb} = ion thruster efficiency in hybrid scheme
$\eta_t(\eta_{ep})$ = tether (electrical) propulsive efficiency
μ = ion thruster to Lorentz thrust ratio
ρ = tether density
σ_c = tether conductivity
τ = mission duration
τ_{ep} = characteritic time of electric propulsor

I. Introduction

ELECTRIC propulsion is more efficient than chemical propulsion for long-duration missions requiring low thrust. In turn, electrodynamic (ED) tethers are much more efficient than ion thrusters for much longer times; in low Earth orbit (LEO), this can more than balance the facts that tether thrust exhibits low pointing accuracy and is ambient-plasma dependent, which are not critical for the performance of ED tethers, involving slow, average operation.

The standard ED tether ejects electrons at a cathodic end, typically through a hollow cathode (HC), and collects electrons passively, either at the anodic end, as in the tethered satellite system $1R$ mission,[1] or over an anodic segment left bare of insulation.[2] Use of an HC as active anode was tested in the plasma motor generator (PMG) mission.[3] It has now been suggested that, instead of collecting electrons, ions could be ejected at the anodic tether end by an ion thruster, which, furthermore, would add to the propulsive capability of the system.[4]

In regular ion thrusters, electrons from an HC are also emitted immediately downstream in the ion beam to limit space-charge effects that affect thruster performance. In the suggested scheme, with ion thruster and its HC operating at opposite tether ends, and the neutralizing current that flows along the tether giving rise to Lorentz thrust, the thruster would need to operate effectively without neutralizer (lying kilometers away). Hopefully, any reduction in propulsive efficiency might be small in new-generation ion thrusters for space exploration, which will have lower beam density and higher exhaust velocity (higher specific impulse).

The suggested ED tether/ion thruster hybrid system, combining ED-tether propulsion and a modified electrostatic propulsion, was said to merge the propellantless propulsion capability of ED tethers and the efficient propulsion capability of electric propulsion, with any increase in mass with respect to the ion thruster, or the ED tether, working alone, to be assessed on a case-by-case basis depending on the application.[4] A generic such assessment is carried out here.

II. Impulse-to-Mass Ratio in Space Propulsion

The simpler figure of merit for space thrusters is the ratio between total impulse required by the mission and mass of the system dedicated to thrusting; that ratio should be maximum. Missions that may be characterized by a total impulse $F\tau$ are reboost of the International Space Station and space-tug operations. For a chemical rocket, thrust F is $\dot{m}c_{ch}$, and mass is basically propellant mass $\dot{m}\tau$, ignoring, for simplicity, a correction from tankage and plumbing. One then has

$$\frac{\text{total mission impulse}}{\text{mass of thrusting system}} = \frac{F\tau}{\dot{m}\tau} = c_{ch} \equiv g_0 \times \text{specific impulse} \tag{1}$$

The greater the specific impulse is the better, but the exhaust or jet velocity is clearly limited in case of chemical combustion.

Greater exhaust velocities can be achieved through electric propulsion, which makes use, however, of a power plant that adds to the mass of the system. With a supply power

$$W_s = Fc_{ep}/2\eta_{ep} = \dot{m}c_{ep}^2/2\eta_{ep} \tag{2}$$

one finds [5,6]

$$\frac{\text{mission impulse}}{\text{system mass}} = \frac{F\tau}{\dot{m}\tau + \alpha W_s} = \frac{c_{ep}\tau}{\tau + \alpha c_{ep}^2/2\eta_{ep}} \Rightarrow c_{ep} \times \frac{\tau}{\tau + \tau_{ep}} \tag{3}$$

$$\tau_{ep} \equiv \alpha c_{ep}^2/2\eta_{ep} \approx 1.14 \times \alpha(\text{kg/kW}) \times (c_{ep}/30\ \text{km/s})^2 \times 0.65/\eta_{ep}\ \text{weeks} \tag{4}$$

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with $\alpha \sim 5$–25 kg/kW typically. State-of-the-art values for ion thrusters are $c_{\text{ep}} \sim 30$ km/s (over 10 times greater than in chemical rockets) and $\eta_{\text{ep}} \sim 0.65$; corresponding values for Hall thrusters are $c_{\text{ep}} \sim 15$ km/s and $\eta_{\text{ep}} \sim 0.50$ (Ref. 7).

Only for $\tau \gg \tau_{\text{ep}}$, however, does the impulse-to-mass ratio in Eq. (3) reaches up to the value of exhaust velocity c_{ep}. Clearly, one cannot discuss the advantages of electric vs chemical propulsion without reference to mission duration (independently of thrust level, although long missions will usually require low thrust). This is already manifest in the Tsiolkovsky equation for the standard mission requiring a thruster to impart some Δv to a payload, which, in the case of electric propulsion, reads (see Ref. 8)

$$\Delta v = c_{\text{ep}} \times \ell n \left| \frac{1 + (\tau_{\text{ep}}/\tau)}{\text{payload mass ratio} + (\tau_{\text{ep}}/\tau)} \right| \tag{5}$$

Again, we have $\Delta v / c_{\text{ep}}$ very small for $\tau \ll \tau_{\text{ep}}$.

III. Impulse-to-Mass Ratio of a Bare Tether

An ED bare tether uses no propellant, and the HC-expellant mass, which is two to three orders of magnitude smaller than propellant mass consumed by an ion thruster for the same mission impulse, can be fully ignored.[5,6] One then has

$$\text{system mass} \approx \alpha W_s + \alpha_t \times \rho A L \tag{6a}$$

$$F = I_{\text{av}} L B_\perp \tag{6b}$$

where the factor $\alpha_t \sim 2$-3 accounts for tether-related hardware (deployer/ballast mass). One then finds[5,6]

$$\frac{\text{mission impulse}}{\text{system mass}} = \frac{I_{\text{av}} L B_\perp \tau}{\alpha W_s + \alpha_t \rho A L}$$

$$= \frac{\eta_{\text{eff}} \tau}{\alpha U_{\text{orb}}} \left(\frac{1}{\eta_{\text{eff}}} \equiv \frac{1}{\eta_t} + \frac{1}{\tilde{E}_m^2 i_{\text{av}}} \right) \tag{7}$$

where $\eta_t W_s \equiv W_m = I_{\text{av}} E_m L$, $\tilde{E}_m = E_m \sqrt{(\alpha \sigma_c / \alpha_t \rho)}$ (typically around unity for aluminum tethers), $E_m = U_{\text{orb}} B_\perp$, and $i_{\text{av}} = I_{\text{av}} / \sigma_c E_m A$. Note that η_{eff} appears as an effective or overall efficiency that takes into account tether hardware as part of the power plant.

In general, both η_t and i_{av}, and thus η_{eff}, take values less than (but of order) unity that derive from detailed bare-tether analysis, showing greater efficiency η_{eff} if only some lower tether segment of length L_{bt} is left bare.[5,6] When small HC and ionospheric circuit-closure impedances are neglected, both η_t and i_{av} depend on dimensionless numbers L_{bt}/L^*, L^*/L, and $W_s/\sigma_c E_m^2 A L$, where L^* is a length gauging bare-tether collection impedance against ohmic resistance,[2,5,6]

$$L^* \equiv \frac{(m_e E_m)^{\frac{1}{3}}}{2^{\frac{7}{3}} e} \left(6\pi \frac{\sigma_c}{N_0} \times \frac{A}{p} \right)^{\frac{2}{3}} \approx 4.2\ \text{km} \times \left(\frac{E_m}{0.1\ \text{V/m}} \right)^{\frac{1}{3}}$$

$$\times \left(\frac{A}{p \times 0.1\ \text{mm}} \frac{10^{11}\ \text{m}^{-3}}{N_0} \right)^{\frac{2}{3}} \tag{8}$$

for aluminum. For optimal conditions $L_{\text{bt}} \sim L^* \ll L$ (easier to satisfy for tape tethers), one has[5,6]

$$1/\eta_{\text{eff}} \approx 1 + i_{\text{av}} + \left(1 \big/ \tilde{E}_m^2 i_{\text{av}} \right) \tag{9}$$

showing a minimum, $1 + 2/\tilde{E}_m$, for a ratio $W_s/\sigma_c E_m^2 A L$ such that $i_{\text{av}} = 1/\tilde{E}_m$.

One can rewrite Eq. (7) as

$$\frac{\text{mission impulse}}{\text{system mass}} = c_{\text{ep}} \frac{\tau}{\tau_{\text{ep}}} \times \frac{\eta_{\text{eff}} c_{\text{ep}}}{2 \eta_{\text{ep}} U_{\text{orb}}} \tag{10}$$

With $c_{\text{ep}} \sim 30$ km/s, $U_{\text{orb}} \sim 7.5$ km/s, and $\eta_p \sim 0.65$, the last fraction in Eq. (10) is approximately unity for a typical value $\eta_{\text{eff}} \sim \frac{1}{3}$. Comparing now Eqs. (3) and (10) clearly shows a much higher impulse-to-mass ratio for the ED tether in the case $\tau \gg \tau_{\text{ep}}$; they are comparable in the case $\tau \sim \tau_{\text{ep}}$.

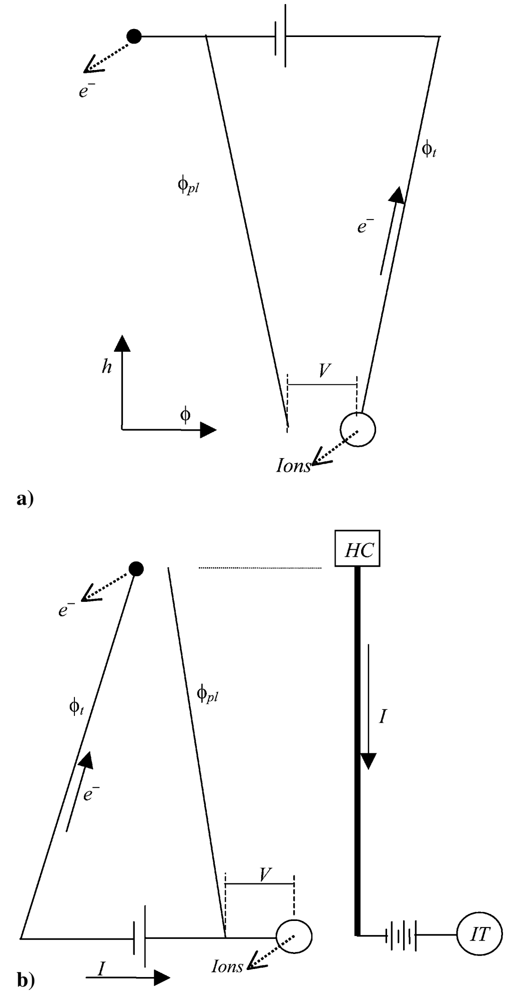

Fig. 1 Profiles of tether and plasma potentials vs distance h from bottom, with power supply at a) top and b) bottom.

IV. Impulse-to-Mass Ratio of an ED Tether/Ion Thruster System

There are basically two possible arrangements for this hybrid or coupled system, with the entire tether insulated: either having the power supply at the top (Fig. 1a) or at the bottom (Fig. 1b). Note that only the first arrangement would make the tether positive relative to the plasma and, thus, operate as bare tether. For either Fig. 1a or Fig. 1b, one has a power supply

$$W_s = V I + E_m L I + Z I^2$$

with $Z = L/\sigma_c A$ and $\eta_{\text{hb}} \times V I = \frac{1}{2} \dot{m} c_{\text{ep}}^2$.

We then have

$$\text{system mass} \approx \dot{m}\tau + \alpha W_s + \alpha_t \rho A L \tag{11a}$$

$$F = \dot{m} c_{\text{ep}} + I L B_\perp \tag{11b}$$

yielding

$$\frac{\text{mission impulse}}{\text{system mass}} = \left\{ (\dot{m} c_{\text{ep}} + I L B_\perp)\tau \Bigg/ \left[\dot{m}\tau + \alpha \left(\frac{\dot{m} c_{\text{ep}}^2}{2\eta_{\text{hb}}} + E_m L I + \frac{L I^2}{\sigma_c A} \right) + \alpha_t \rho A L \right] \right\}$$

$$\Rightarrow c_{\text{ep}} \times \frac{(1+\mu)\tau}{\mu(\tau + \tau_{\text{ep}} \times \eta_{\text{ep}}/\eta_{\text{hb}}) + \tau_{\text{ep}} \times 2\eta_{\text{ep}} U_{\text{orb}} / \eta_{\text{eff}}\, c_{\text{ep}}} \tag{12}$$

where we introduced the thrust ratio $\mu \equiv \dot{m} c_{\text{ep}} / I L B_\perp$, wrote

$$1 + i + 1 \big/ \tilde{E}_m^2 i \equiv 1/\eta_{\text{eff}} \tag{13}$$

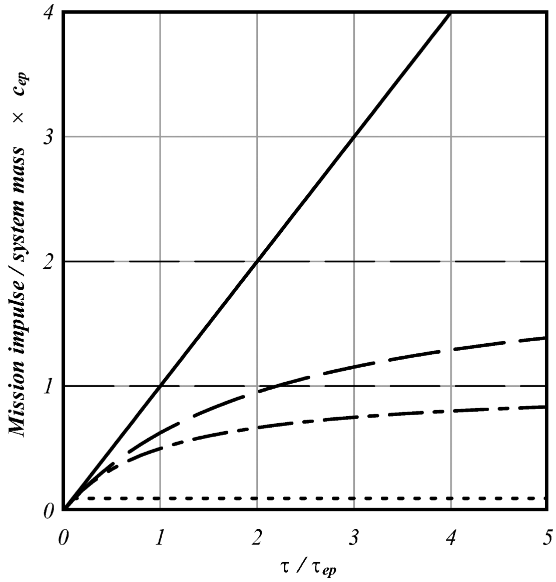

Fig. 2 Normalized equivalent specific impulse vs normalized time: - - - -, chemical; – – –, electrical; – – –, hybrid; and ——, tether propulsion.

and allowed for a reduced propulsive efficiency of the ion thruster working without local neutralizing current, $\eta_{hb} < \eta_{ep}$.

The case $\mu \to \infty$ ($\mu = 0$) corresponds to an ion thruster (an ED bare tether) working by itself, Eq. (12) recovering results in Eq. (3) [in Eq. (10)], whereas it takes the form

$$\frac{\text{mission impulse}}{\text{system mass}} = c_{ep} \times \frac{2\tau}{\tau + \tau_{ep}\eta_{ep} \times \left(\eta_{hb}^{-1} + 2U_{orb}/\eta_{eff}c_{ep}\right)} \tag{14}$$

for a middle hybrid system, $\mu = 1$. Figure 2 shows all four expressions in Eqs. (1), (3), (10), and (14); in Fig. 2, we set $c_{ch}/c_{ep} = 0.1$, $\eta_{ep}/\eta_{hb} = 1.2$, and $2\eta_{ep}U_{orb}/\eta_{eff}\, c_{ep} = 1$. The ED tether is the clear winner. Note that taking into account the small HC-expellant mass it consumes would make its straight line in Fig. 2 finally reach a horizontal asymptote at 10^2–$10^3 \gg 1$; also, a higher c_{ep} in future ion thrusters will enhance the advantage of the plain ED tether, although it will increase τ_{ep} as well.

In the example discussed in Ref. 4, the ion thruster had specific impulse = 4000 s, $F = 0.2$ N, $W_s = 5.7$ kW, and $1/\alpha = 48$ W/kg, corresponding to $c_{ep} \approx 40$ km/s, $\eta_{ep} = 0.7$, and $\tau_{ep} \approx 39$ weeks. The tether was an aluminum wire with $A = \pi$ mm^2, $L = 7$ km, and current and Lorentz force taken as $I = 3.2$ A and $ILB_\perp = 0.44$ N. This yields $\mu = 0.45$, $B_\perp \approx 0.2$ G, $E_m \approx 0.15$ V/m, and finally $i \approx 0.194$, $\tilde{E}_m \approx 1.774 \Rightarrow \eta_{eff} \approx 0.35$. (We set $\alpha_t = 2$, making $\sqrt{(\alpha_t \rho/\alpha\sigma_c)} \approx 0.086$ V/m.) Equation (12) here reads

$$\frac{\text{mission impulse}}{\text{system mass}} = \frac{1.45\tau \times 40\,\text{km/s}}{0.45\tau + 1.29\tau_{ep}} \tag{15}$$

An (equivalent) specific impulse ≈ 128 km/s was ascribed to this hybrid system in Ref. 4. Note, however, that that is only the limit of Eq. (15) for $\tau \gg 2.9\tau_{ep}$, the ($\mu = 0$) ED tether's impulse-to-mass ratio then being $155\,\text{km/s} \times (\tau/2.9\tau_{ep}) \gg 128$ km/s. Corresponding values for $\tau = \tau_{ep}$ are $\sim$33 km/s for the hybrid system vs 53 km/s for the plain ED tether. This should be a thin tape rather than a round wire, tape width $= \pi$ cm and thickness $= 0.1$ mm, keeping mass (and resistance) and yielding $L^* \sim 3\,\text{km} \times (10^{11}\ \text{m}^{-3}/\text{N}_0)^{2/3}$. Although condition $L^*/L \ll 1$, and, thus, Eq. (9), will fail at night, a value $\eta_{eff} \sim \frac{1}{3}$ could be expected because the current $i \approx 0.19$ is far from optimal in either Eq. (13) or Eq. (9) with $\tilde{E}_m \approx 1.77$; a power W_s yielding $i = 1/\tilde{E}_m \approx 0.56$ gives $\eta_{eff} \approx 0.47$.

V. Conclusions

We have shown that fully taking into account all system masses, and giving due consideration to mission duration, the suggested hybrid system is substantially outperformed by ED tethers for their long-mission niche and performs similarly to ED tethers and ion thrusters otherwise. The point here is that coupling an ion thruster to an ED tether increases the impulse-to-mass ratio of the ion thruster but decreases that ratio for the ED tether (enormously so in case of very long missions). Independently, the hybrid system appears as much more complex than either ion thruster or ED tether operating by itself.

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